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Damped least square based genetic algorithm with Ggaussian distribution of damping factor for singularity-robust inverse kinematics

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Abstract

Robot inverse kinematics based on Jacobian inversion encounters critical issues of kinematic singularities. In this paper, several techniques based on damped least squares are proposed to lead robot pass through kinematic singularities without excessive joint velocities. Unlike other work in which the same damping factor is used for all singular vectors, this paper proposes a different damping coefficient for each singular vector based on corresponding singular value of the Jacobian. Moreover, a continuous distribution of damping factor following Gaussian function guarantees the continuous in joint velocities. A genetic algorithm is utilized to search for the best maximum damping factor and singular region, which used to require ad hoc searching in other works. As a result, end effector tracking error, which is inherited from damped least squares by introducing damping factors, is minimized. The effectiveness of our approach is compared with other methods in both non-redundant robot and redundant robot.

Keywords: Kinematic singularities; Damped least squares; Gaussian function genetic algorithm

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1. Introduction

The robot inverse kinematics at the level of velocity solves the robot joint velocities with a given robot end-effector velocity. The relation between robot joint space and robot end-effector Cartesian space is written through a robot Jacobian matrix. In this approach, a kinematic singularity is a critical issue that has to be resolved. At singularities or a singular configuration of a robot, the Jacobian loses its rank and thus, the robot loses its degree of freedom. In other words, the robot end-effector is unable to generate velocity in some direction. However, it is more important to monitor the vicinity of a singular configuration than examining the value of the determinant. In the neighborhood of the singular configuration, the classical inverse of the Jacobian results in very high joint velocities [1]. Physically, those joint velocities are not achievable and result in inaccurate motion of the end-effector. Moreover, such motions are harmful to the environment, the user and the robot itself.

One of approaches to control robot through singularities proposed in [2, 3] are based on damped least square (DLS) method. This method is also known as Levenberg-Marquardt stabilization method. The idea is to weight the minimized property between end-effector tracking error and norm of joint velocities. Hence, it creates continuity in transition between singular configuration and non-singular configuration. The key point in applying DLS is the selection of the damping factor. In the past, a constant damping factor was proposed in [2]. Nakamura suggested selecting a damping factor as a function of manipulability measure [3]. Because the manipulability is defined based on robot Jacobian, it is thus an index to indicate the closeness to singular configuration of a manipulator.

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However, manipulability is not an accurate index to measure the proximity to singularities [4] and cannot be calculated real time with a complex system due to expensive computation. Later, Maciejewski and Klein [4] proposed another method to choose damping factor based on the smallest singular value, which is the most appropriate measure of neighborhood of singularity. In that work, the smallest singular value is not calculated by singular value decomposition (SVD) because of the computational cost. Instead, Maciejewski and Klein proposed an effective way to estimate the smallest singular value. Another work also based on smallest singular value to choose the damping factor for an common type of industrial robot was introduced by Chiaverini [5]. A more general approach in selecting the damping factor was proposed by Deo and Walker in 1992 [6]. They defined an optimization problem to minimize the end effector tracking error subject to joint velocity norm with respect to the maximum one that the manipulator can achieve. Thus, the threshold is a specifically physical characteristic of the manipulator. However, this approach is not appropriate in real-time application due to the calculation of an optimization problem at each iteration. Recently, Buss and Kim proposed an extension of the damped least square method, which is called selectively damped least square (SDLS) method, todeal with the robot motion near singularities [7]. The method has the advantage in selecting the damping factor by considering the relative of the end-effector position and the target position. The selectively damped least square method, however, shows a better performance than DLS only near the target position.

Based on DLS concept, our paper proposes firstly a technique to generate the damping factor, which is based on Gaussian functions, for each singular vector in order to guarantee continuity of joint velocities as well as reduce end-effector tracking error in section 3. Moreover, a genetic algorithm presented in section 4 is implemented to obtain best DLS based parameters by doing optimization between joint velocities and damping factors. Thus, this technique does not only make the implementation of the DLS method more convenient in choosing related parameters (singular region and maximum damping factor) but it also reduces the tracking error. The combination of the two techniques allows a more accurate trajectory of the end-effector and prevents an ad hoc selection for the best related parameters in DLS.

2. Inverse kinematics based on damped least squares

There are several ways to solve inverse a kinematic problem numerically based on the robot Jacobian matrix such as Jacobian transpose, pseudo inverse of Jacobian and the damped least square method [7, 8]. The most common way to solve inverse kinematic is by using the pseudo inverse of the Jacobian matrix to solve the robot joint velocities $\dot{q}$ with a given robot end-effector velocity $\dot{X}$. This method was proposed by Whitney [1].

$$\dot{q} = J^{+}\dot{X} \tag{1}$$

Where J^{+} is the pseudo inverse of the robot Jacobian J. This method gives the best possible solutions that minimize joint velocity norm $\|\dot{q}\|^2$ and end-effector tracking error $\|\dot{X} - J\dot{q}\|^2$. Unfortunately, the pseudo inverse method is unstable near singularities. This occurs when there is a direction of movement of the end effectors that is not (first-order) achievable by changes in joint angle [7]. The increase of joint velocities near the singularity can be explained in terms of SVD [9]. By SVD theory, with a matrix J ($m \times n$) ranked r. there exist orthogonal matrices U and V of dimensions $m \times n$ and $n \times n$ respectively, such that [8]:

$$J = U \Sigma V^T \tag{2}$$

Where $\Sigma = \begin{bmatrix} S_{r\times r} & 0_{r\times(n-r)} \\ 0_{(m-r)\times r} & 0_{(m-r)\times(n-r)} \end{bmatrix}$ and S is a diagonal matrix formed by the non zero singular values of J, which are arranged in descending order, i.e $\sigma_1 \geq \sigma_2 \geq \cdots \sigma_r$. Hence, the pseudo inverse of the Jacobian (full rank) could be computed as follows.

$$J^{+} = \sum_{i=1}^{m} \frac{1}{\sigma_i} V_i U_i^T \tag{3}$$

Where σ_i is the singular value of J.

Finally, joint velocities in (1) can be expressed in the following form:

$$\dot{q} = \sum_{i=1}^{m} \frac{1}{\sigma_i} V_i U_i^T \dot{X} \tag{4}$$

While a robot approaching a singular configuration, the smallest singular value reaches zero and it causes infinite joint velocities as implied in (4). Hence, the norm of joint velocities and the accuracy of the solution of Eq. (1) are considered in DLS in order to achieve a smooth motion near singularities [2, 3]. In other words, DLS finds the solution to minimize the quantity $\left\|J\dot{q}-\dot{X}\right\|^2+\lambda^2\left\|\dot{q}^*\right\|^2$, where $\lambda = 0$ is a damping factor and $\dot{q}^*$ specify joint velocities in DLS solution. This solution is typically obtained as the least squares solution of the following system [8].

$$\begin{bmatrix} J \\ \lambda I_n \end{bmatrix}\dot{q}^* = \begin{bmatrix} \dot{X} \\ 0_{n\times 1} \end{bmatrix} \tag{5}$$

or

$$\begin{bmatrix} J \\ \lambda I_n \end{bmatrix}^T \begin{bmatrix} J \\ \lambda I_n \end{bmatrix}\dot{q}^* = \begin{bmatrix} J \\ \lambda I_n \end{bmatrix}^T \begin{bmatrix} \dot{X} \\ 0_{n\times 1} \end{bmatrix}$$

The above system gives the solution:

$$(J^T J + \lambda^2 I_n)\dot{q}^* = J^T \dot{X}$$

It is equivalent to [4]:

$$\dot{q}^* = J^T (J^T J + \lambda^2 I_m)^{-1} \dot{X} \tag{6}$$

These joint velocities can be derived further by the application of SVD:

$$\dot{q}^* = \sum_{i=1}^{m} \frac{\sigma_i}{\sigma_i^2+\lambda^2} V_i U_i^T \dot{X} \tag{7}$$

The stabilization of DLS near singularities now can be clearly explained [2]. When the robot operates far from singular configuration and a suitable damping factor given so that $\sigma_i \gg \lambda$, then $\frac{\sigma_i}{\sigma_i^2+\lambda^2} \approx \frac{1}{\sigma_i}$. With the given damping factor, the robot approaching singular configuration also means $\sigma_i \ll \lambda$ then $\frac{\sigma_i}{\sigma_i^2+\lambda^2} \approx \frac{\sigma_i}{\lambda^2} \to 0$. It means that the damping factor limits the norm of joint velocities near singularities [8].

3. Gaussian distribution of damping factor

The damping factor depends on robot structure, target positions and must be chosen carefully to make DLS numerically stable [7]. The reason is the damping factor creates a trade-off between the precision of the solution and the increase in joint velocities near singularities. The error due to damping factor in Cartesian velocity space is given as follows.

$$\dot{e}_x = J\dot{e}_q = J(\dot{q}-\dot{q}^*) = \sum_{i=1}^{m} \frac{\lambda^2}{\sigma_i^2+\lambda^2} U_i U_i^T \dot{X} \tag{8}$$

The successful selection of a damping factor requires that the damping factor is small enough to not create Cartesian error by damping as shown in (8) [8] and can be calculated online based on appropriate index of singular proximity. This paper proposes a new method to select the damping factor based on Gaussian function and the singular value of the robot Jacobian.

The advantages of Gaussian distribution to damping factor, which is formulated below, are two-fold. Firstly, this function makes a continuous transition from un-damped phase to damped phase. In other words, outside the area of singularities, damping factor varies continuously around zero and increases exponentially when approaching a singularity region. Whenever the robot passes through singularities, the damping factor decreases exponentially to zero. Hence, this method is different from all other methods, where the damping factor is chosen by piecewise functions [3-5, 10-12]. The piecewise function originally creates discontinuities in applying the damping factor. Consequently, it results in a discontinuity of joint velocities if the damping factor following piecewise function is applied to each singular vector. Therefore, our proposal guarantees smooth motion of the robotic system when it is approaching singularities even if different damping factors are used for singular vectors. Secondly, a damping factor is selected for each singular vector. Hence, damping is imposed on only a singular vector, which is ill conditioned. Although the decision of choosing a damping factor based on only the smallest singular value in other works makes the Jacobian matrix numerically stable, it creates unnecessary damping for other singular vectors, which are not ill conditioned. Thus, more tracking error is produced in other methods and the difference between the actual and the desired

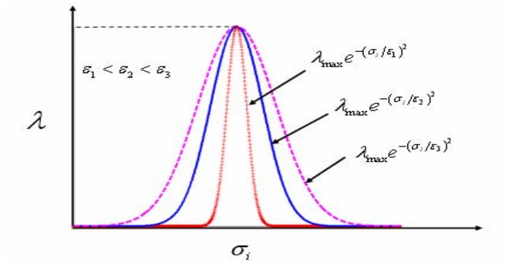

Fig. 1. The proposed distribution of damping factor with different singular region ε .

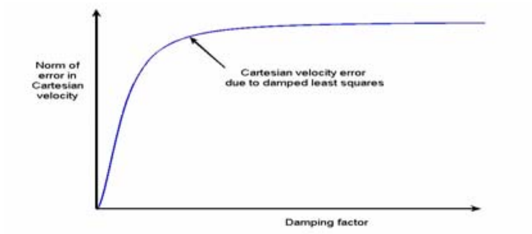

Fig. 2. Norm of Cartesian error due to damped least squares versus damping factor with fixed singular values and end-effector velocity.

trajectory is consequently larger. The damping factor in our approach is suggested as follows.

$$\lambda_{Gi} = \lambda_{\max} \cdot e^{-(\sigma_i/\varepsilon)^2} \tag{9}$$

Where λ_{Gi} is the damping factor for the singular vector i following Gaussian function, and $\lambda_{\max}$ is the maximum value of the damping factor, ε is a scalar quantity indicating region of singularities and σ_i is singular value. Obviously, $\lambda_{\max}$ contributes to the damping magnitude or the height of the Gaussian distribution. On the other hand, ε is compared with the singular value of each singular vector to indicate whether the robot is approaching a singular configuration or not (Fig. 1.). Thus, it determines the period that damping is applied to system.

The small selection of a singular region leads to the algorithm not catching singularities and does not provide damping to the system. In contrast, unnecessary damping is applied when a large singular region is defined. The two parameters $\lambda_{\max}$ and ε are chosen with a trial-and-error approach in all other works or by hard constraint of velocities [4]. However, those two parameters can be selected optimally as proposed in the next section.

4. Searching optimal maximum damping factor and singular region by genetic algorithm

An optimal selection of the maximum value of the damping factor and singular region in (9) contributes to the successful application of the damped least square method not only in our work but also in other approaches. The choice of maximum value of damping factor is somewhat easier than that of singular region since it purely depends on how small the singular values are. The singular region should be defined to be close to all possible singularities. However, it is noted that singular configurations are not necessary to be known with our approach. Because once singular configurations are known, it is easy to overcome them by stopping the joints, which causes ill condition in the Jacobian. If there is a constraint in the tracking error due to the introduction of a damping factor, a maximum damping factor could be found by solving nonlinear Eq. (8) once the end-effector velocity is known. However, that approach cannot be applied to real-time control due to expensive computation in solving the nonlinear Eq. (8) on-line. The norm of tracking error at end-effector, which occurs by using damped least squares, and damping factor possess monotonic behavior as shown in Fig. 2.

In this paper, the constraint of maximum allowable joint velocity is used to find the optimal maximum damping factor $\lambda_{\max}$ and singular region ε . The goal is to find the two parameters so that the Cartesian velocity error due to damped least squares is kept minimum. Hence, the actual trajectory of the end effector is kept as close as possible to the desired trajectory while the robot passes through a singular configuration. The trade off in increasing joint velocity is limited by constraint of maximum allowable joint velocity [4, 6]. This optimization problem is formulated as follows and is implemented with genetic algorithm.

$$\begin{aligned} &Min\ \|\dot{e}_X\| \\ &Subject\ to\ \|\dot{q}^*\| \le \Delta \\ &\qquad\qquad \varepsilon > \sigma \end{aligned} \tag{10}$$

Where $\|\dot{e}_X\|$ is the norm of Cartesian velocity error due to damped least squares. Damping factor in this case follows Gaussian form as proposed in (9). Robot end-effector velocity $\dot{X}$ could be fixed or could be varied in a range as in visual servoing application. The singular value σ in the implementation here is given in a range near zero. This is not a strict range from the Jacobian matrix but a large range that it put the genetic algorithm working in the possible singular region. The matrix U is predetermined with a configuration corresponding to a small singular value or singular configuration of robot. The norm of Cartesian velocity error is derived from Eq. (8) [8].

$$\|\dot{e}\| = \sqrt{\sum_{i=1}^{m}\left(\frac{\lambda_{Gi}^2}{\sigma^2+\lambda_{Gi}^2}\right)^2 (U_i^T \dot{X})^2} \tag{11}$$

The constraint is that the joint velocities in (7) should be less than maximum allowable joint velocity Δ. This constraint comes from the physical specification of each robot in joint velocities and joint acceleration. The norm of joint velocities can be derived from Eq. (7) as follows (8).

$$\|\dot{q}^*\| = \sqrt{\sum_{i=1}^{m}\left(\frac{\sigma_i}{\sigma_i^2+\lambda_{Gi}^2}\right)^2 (U_i^T \dot{X})^2} \tag{12}$$

It is noted that the selection of the optimal maximum damping factor λ_{max} and singular region ε by genetic algorithm is performed only one time before inverse kinematics is run. Hence, the consumption of time by genetic algorithm does not cause any delayed problem in running the inverse kinematic algorithm. Moreover, both object function and constraints are nonlinear functions. It is important to recall that the Jacobian matrix in those functions is very close to singularities. Thus, by using genetic algorithm, we can use its advantages in dealing with ill-condition of optimization function and there turn of global optimum. The optimal result of the two parameters is then applied to DLS with damping factor following Gaussian function. Therefore, the overall result is minimum Cartesian error due to introduction of damping factor.

5. Simulation results

Genetic algorithm used in our application is based on binary code. In addition, because there is a trade off between joint velocity and tracking error, the solutions of our problem, which are λ_{max} and ε, have high chance to locate near maximum allowable joint velocity. Hence, the most appropriate technique to deal with inequality constraint in the genetic algorithm of our problem is penalty technique [13]. Moreover, other parameters which are used in our case are shown in the table below.

Table 1. Parameters of genetic algorithm used to obtain maximum damping factor and singular region.

Parameters	Values
Population size	300
Crossover fraction	0.9
Mutation function	Adaptive
Selection function	Roulette

Table 2. Denavit Hartenberg table of ABB IRb 2000 manipulator.

Link	α	a	d	θ
1	0	0	0	$\pi/2$
2	$\pi/2$	0	0	$\pi/2$
3	0	0.710	0	$\pi/2$
4	$\pi/2$	0.125	0.850	0
5	$\pi/2$	0	0	0
6	$\pi/2$	0	0.100	0

Once the two parameters λ_{max} and ε are obtained from the genetic algorithm, inverse kinematics based on damped least squares with Gaussian distribution of damping is performed. Several methods of damped least square based inverse kinematics are compared in order to show our effectiveness.

5.1 Non-redundant robot study case

The algorithm is firstly applied to non-redundant robot ABB IRb 2000, a six degree of freedom robot. Parameters of the robot are shown in table 2 [5]. As mentioned in section 4, the singular value for the genetic algorithm is set between the interval 0.0 and 0.07. This range shows a rough estimation of the vicinity of singularities. The maximum allowable norm of joint velocity $\Delta = 0.8(rad/s)$ is used for this non-redundant case.

Trajectory 1: Several approaches based on damped least squares are compared to go through a wrist singularity of this robot. The trajectory through the wrist singularity begins with the initial configuration $q = (0\ \ \pi/12\ \ -\pi/2\ \ 0\ \ 0.15\ \ 0)^T rad$ and an increment $\Delta p = (0.18\ \ 0.45\ \ -0.45)^T m$ [5]. Our approach, Gaussian based damped least squares (G-DLS), is compared with the basic approach of damped least squares (DLS) with a constant damping factor 0.09 and the others methods called M-DLS and C_DLS [5] in Fig. 3 and Fig. 4. M-DLS is also Gaussian based DLS but the damping factors are similar to all singular vectors. With this robot, both G-DLS and M-DLS method operate with the same parameters which

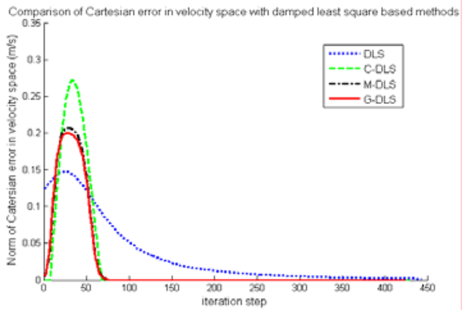

Fig. 3. A comparison of norm of Cartesian error velocity due to damped least squares on trajectory 1 of non-redundant case.

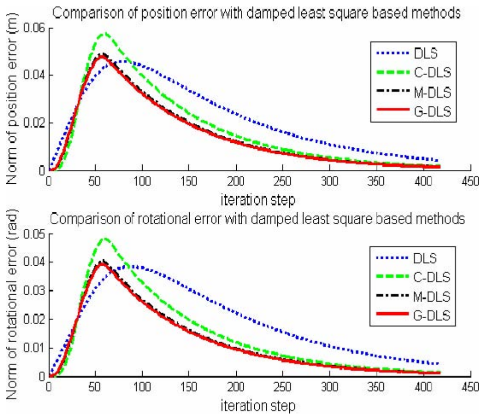

Fig. 4. A comparison of Cartesian error (norm of position error and norm of rotational error) due to damped least squares on trajectory 1 of non-redundant case.

are obtained from genetic algorithm, $\lambda_{\max} = 0.09$ and $\varepsilon = 0.037$. C-DLS is the method proposed in which also uses maximum damping factor and singular region as parameters in a piece wise function and sets $\lambda_{\max} = 0.04$ and $\varepsilon = 0.04$ in this simulation.

In Fig. 3, it is shown that the error velocity of our approach has a smooth transition from outside of the singular region into that of inside. The reason is that the damping follows a smooth transition through Gaussian function and results in Cartesian error velocity. On the other hand, in Fig. 4, both norm of position error and rotational error in our approach are smaller than those of other approaches. Although all extended methods based on damped least square have the peak of error higher than the basic approach, their errors are reduced much faster than the basic approach. With our approach, high joint velocities with Jacobian pseudo inverse method, shown in Fig. 5, are damped to reasonable joint velocities in Fig. 6.

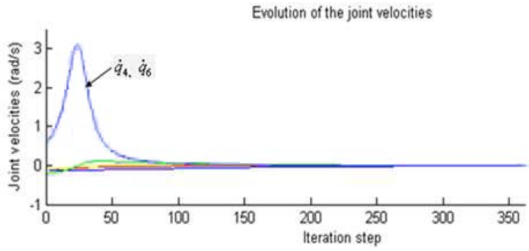

Fig. 5. Joint velocities through wrist singularity of ABB IRb 2000 with Jacobian pseudoinverse method on trajectory 1 of non-redundant case.

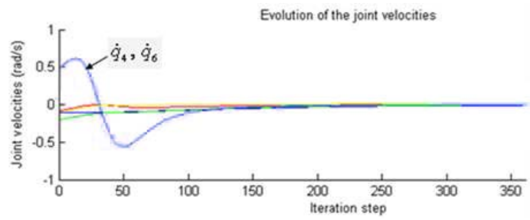

Fig. 6. Joint velocities through wrist singularity of ABB IRb 2000 with G-DLS on trajectory 1 of non-redundant case.

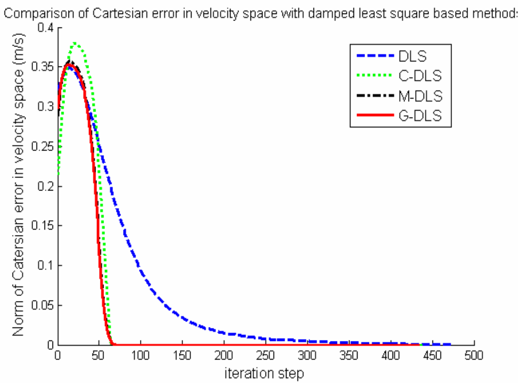

Fig. 7. A comparison of norm of Cartesian error velocity due to damped least squares on trajectory 2 of non-redundant case.

Trajectory 2 : In this trajectory, the initial configuration of the robot is very close to singular configuration. The initial configuration of robot is set at $q = (-0.044\ \ 0.199\ \ -1.613\ \ 0.734\ \ 0.06\ \ 0.734)^T$ and the increment is similar to the one in trajectory 1. Moreover, other parameters are also kept the same as in trajectory 1. Although the initial configuration of robot is close to a singularity, our method still guarantees the quality of damped and keeps the error minimum. The smooth transition of error velocity of our approach is shown in Fig. 7. The algorithm also shows a good performance in keeping small Cartesian error due to DLS as shown in Fig. 8.

Table 3. Denavit Hartenberg table of ultra light-weight Amtec arm.

Link	α	a	d	θ
1	0	0	0	0
2	$-\pi/2$	0	0	0
3	$\pi/2$	0	0.388	0
4	$-\pi/2$	0	0	0
5	$\pi/2$	0	0.349	0
6	$-\pi/2$	0	0	0
7	$\pi/2$	0	0	0

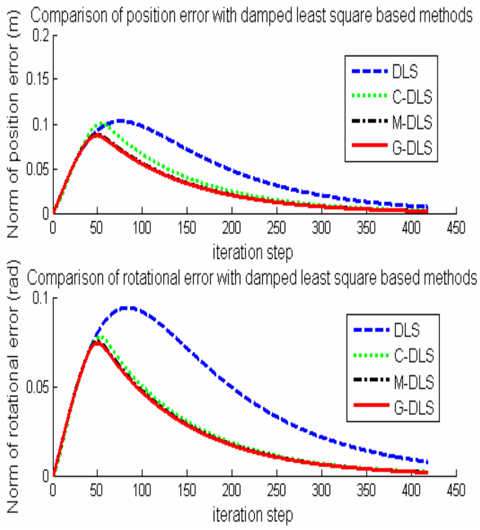

Fig. 8. A comparison of Cartesian error (norm of position error and norm of rotational error) due to damped least squares on trajectory 2 of non-redundant case.

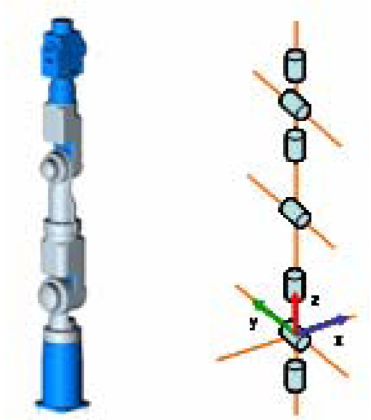

Fig. 9. Ultra light-weight Amtec arm, a seven degree of freedom arm and its kinematics structure.

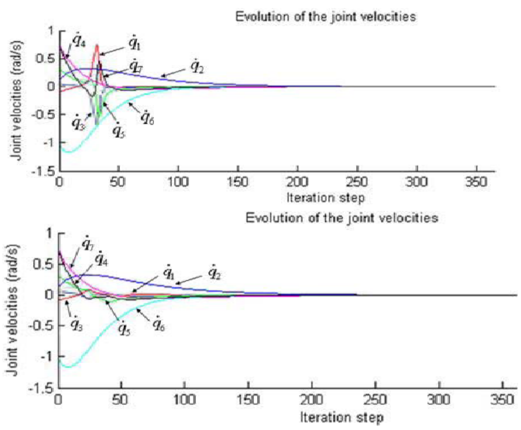

Fig. 10. Double singular configurations by pseudo inverse (upper figure) and response after applying damping (lower figure) on trajectory 1 of redundant case.

5.2 Redundant robot study cases

Our algorithm is also applied to a redundant robot, ultra light-weight Amtec, a seven degree of freedom manipulator which has a kinematic structure like the human arm. The manipulator and its kinematic structure are given in Fig. 9 and its Denavit Hartenberg is given in Table 3. The genetic algorithm in this case also uses parameters given in table 1. The maximum end effector translational velocities and rotational velocities are set 28 cm and 16 degrees per second respectively. Maximum allowable velocity in this redundant case is set at 0.6 rad/s.

Trajectory 1: In Fig. 10, manipulator encounters both a singularity in the second joint and wrist singularity around step 40. The initial configuration $q_{initial} = (0\ -0.349\ 0\ 1.396\ 0\ 0.96\ 0)^T$ and final configuration $q_{final} = (0\ -\pi/6\ 0\ \pi/2\ 0\ -0.61\ 0)^T$ in this trajectory are fixed. With pseudo inverse method, the two singularities cause an increase in the joint velocities of joint 1, 3 and joint 5, 7.

The result which is the burden in related joint velocities is completely eliminated meanwhile the other joint velocities almost have the same form as shown in Fig. 10. In trajectory 1, parameters for C-DLS method are at $\lambda_{\max} = 0.08$ and $\varepsilon = 0.08$. The same maximum damping factor in C-DLS is also used for the basic approach of DLS. The two other methods, M-DLS and G-DLS, are operated with the following parameters obtained by genetics algorithm $\lambda_{\max} = 0.1$ and $\varepsilon = 0.044$. Through the result of simulation, it is shown the G-DLS method generates a smaller Cartesian error due to DLS in both velocity space (Fig. 11) and position or orientation space (Fig. 12) than those of other approaches.

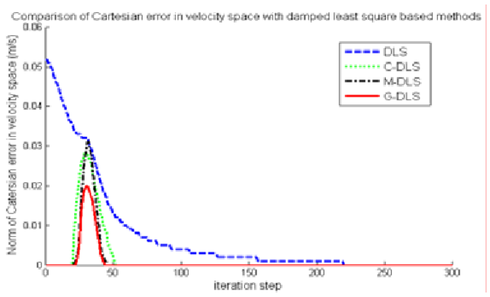

Fig. 11. A comparison of norm of Cartesian error velocity due to damped least squares on trajectory 1 of redundant case.

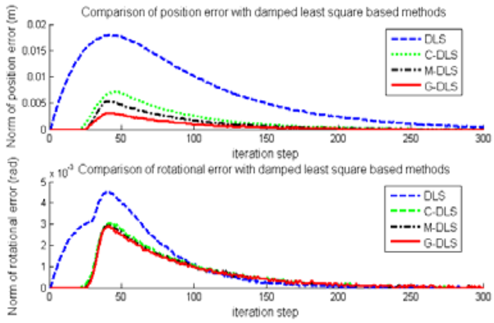

Fig. 12. A comparison of Cartesian error (norm of position error and norm of rotational error) due to damped least squares on trajectory 1 of redundant case.

Trajectory 2 : In order to show effectiveness in creating smaller error on trajectory, especially if the initial configuration is in a singularity region, the initial configuration of trajectory 2 is set as follows: $q_{initial} = (0.059\ \ -0.082\ \ -0.054\ \ 1.710\ \ 0.131\ \ -0.022\ \ 0.149)^T$. The final configuration is the same as the one in trajectory 1. On this trajectory, our method still guarantees a better performance (smaller error due to DLS) than other methods, as shown in Fig. 13 and Fig. 14.

Our approach shows a great effect in reducing error on trajectory, which was caused by the inheritance of the damped least square method. This is the result of strategy to apply different damping on different singular vectors of the Jacobian matrix and optimal parameters obtained from the genetic algorithm. Thus, unnecessary damping is eliminated from singular vectors. The Cartesian errors in velocity space of different approaches are shown in Figs. 3, 7, 11 and 13. Through those figure, G-DLS has the norm of error in velocity space much smaller than that of the basic approach because the damping is applied to defined singular region only. G-DLS also has smaller error velocity than that of M-DLS and C-DLS different damping applied to different singular vector. The

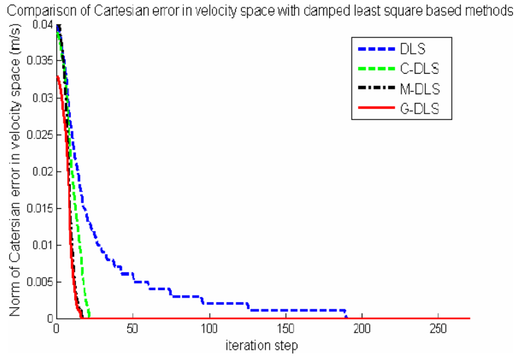

Fig. 13. A comparison of norm of Cartesian error velocity due to damped least squares on trajectory 2 of redundant case.

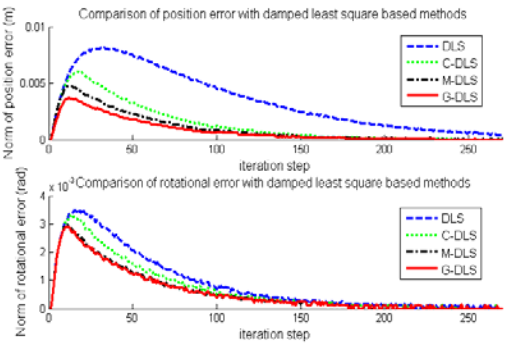

Fig. 14. A comparison of Cartesian error (norm of position error and norm of rotational error) due to damped least squares on trajectory 2 of redundant case.

smaller error in Cartesian velocity space also leads to smaller Cartesian error in both translation error and orientation error in Figs. 4, 8, 12 and 14. Especially, our proposed method provides the two desired characteristics, which are small tracking error and smooth transition between the damped phase and un-damped phase. The two figures characteristics have good performance on both non-redundant and redundant robot.

6. Conclusion

In the problem of Jacobian inversion of robot inverse kinematics, we proposed two strategies based on damped least square method to effectively deal with singularities and minimize error due to the introduction of damping. The first strategy based on Gaussian distribution of damping factor makes a continuous transition from un-damped phase to damped phase and appropriate damping factors for each singular vector. The second one is the implementation of the genetic algorithm to select the optimal parameters used in the first strategy to minimize tracking error, which is caused by the inherited property of damped least square method. The use of genetic algorithm also makes the process of choosing parameters in DLS easier than the current trial-and-error approach. Through simulation with the combination of the two techniques, it is shown that the tracking error is smaller than the basic approach of DLS as well as other extensions of DLS. Moreover, our approach results in smooth transition of error velocity during entering singular region and leaving singular region. The two desired characteristics, the smaller tracking error and the smoothness in error velocity, are shown with starting configuration of robot in side singular region and outside of singular region on two different types of robots.

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Reference

[1] D. E. Whitney, Resolved motion rate control of manipulators and human prostheses, *Man Machine Systems, IEEE Transactions on*, 10 (1969) 47-53.

[2] C. W. Wampler, Manipulator inverse kinematic solutions based on vector formulations and damped least-squares methods, *Systems, Man and Cybernetics, IEEE Transactions on*, 16 (1986) 93-101.

[3] Y. Nakamura, Y. Hanafusa and T. Yoshikawa, Task-priority based redundancy control of a robot manipulator, *The International Journal of Robotics Research,* 6 (1987) 3-15.

[4] A. A. Maciejewski and C. A. Klein, Numerical filtering for the operation of robotic manipulators through kinematically singular configurations, *Journal of Robotic Systems*, 5 (1988) 527-552.

[5] S. Chiaverini, B. Siciliano and O. Egeland, Review of the damped least-squares inverse kinematics with experiments on an industrial robot manipulator, *Control Systems Technology, IEEE Transactions on*, 2 (1994) 123-134.

[6] A. S. Deo and I. D. Walker, Robot subtask performance with singularity robustness using optimal damped least-squares, presented at IEEE International Conference on Robotics and Automation, Nicw, France (1992) 434-441.

[7] S. R. Buss and J.-S. Kim, Selectively damped least squares for inverse kinematics, *Journal of Graphics Tools*, 10 (2005) 37-49.

[8] W. khalil and E. Dombre, Modeling, Identification and Control of Robots, First ed: Hermes Penton Ltd., (2002).

[9] V. Klema and A. Laub, The singular value decomposition: Its computation and some applications, *Automatic Control, IEEE Transactions on*, 25 (1980) 164-176.

[10] S. Chiaverini, Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators, *Robotics and Automation, IEEE Transactions on*, 13 (1997) 398-410.

[11] A. S. Deo and I. D. Walker, Adaptive non-linear least squares for inverse kinematics, presented at International Conference on Robotics and Automation, (1993).

[12] S. Chiaverini, O. Egeland and R. K. Kanestrom, Achieving user-defined accuracy with damped least-squares inverse kinematics, presented at Fifth International Conference on Robots in Unstructured Environments, (1991).

[13] A. R. Conn, N. I. M. Gould and P. L. Toint, A globally convergent augmented lagrangian barrier algorithm for optimization with general inequality constraints and simple bounds, *Mathematics of Computation*, 66 (1997) 261-288.

[14] A. D. Luca, G. Oriolo and P. R. Giordano, Image-based visual servoing schemes for nonholonomic mobile manipulators, *Robotica*, 25 (2007) 131-145.